# MSL Technical Guide 13

# Pressure Gauge Calibration


## Summary

This guide outlines a method to calibrate a pressure gauge and calculate the uncertainty of the gauge corrections. A pooled standard deviation calculation is used to find the gauge repeatability from two complete calibrations. The resulting uncertainty component is obtained with a large number of degrees of freedom justifying the use of a coverage factor $k$ = 2.2 for the calibration. Two worked examples of gauge calibrations are shown.


## Introduction

A practical method is presented to calibrate a gauge, obtain the expanded uncertainty and decide whether the gauge complies with the accuracy requirements of a documentary standard, such as EN 837 [1]. The method involves calibrating the gauge twice and using the results to find the gauge corrections and the uncertainty of the corrections.

The uncertainty calculation has three main components; the reference pressure, the gauge resolution and the gauge repeatability. Other factors like thermal effects, drift or pressure head corrections are small and can be neglected.

Compliance to the accuracy requirements of a standard is determined from the gauge corrections and expanded uncertainty. For example a class 1 gauge, as defined by clause 9.1 of EN837 [1], has a maximum permissible reading error of ±1 % of span. In practise the expanded uncertainty must be taken into account so the maximum error must be less than 1 % of span minus the expanded uncertainty. The maximum permissible error for a class 1 gauge with a span of 4000 kPa is $4000 \times 0.01 = 40$ kPa. If the calibration uncertainty is 25 kPa then all the gauge corrections must be less than $40 - 25 = 15$ kPa to be sure that the gauge complies.

The expanded uncertainty is calculated by multiplying the uncertainty by a coverage factor $k$, see the calibration equation section below. This is done to expand the reported uncertainty interval so it has an estimated level of confidence of 95 % (i.e. there is a 5 % chance the measured value lies outside the interval) [2]. The value of $k$ will depend on the number of repeat measurements (degrees of freedom) made. For three repeat measurements $k = 4.3$, while for ten repeat measurements $k = 2.3$.

Ideally the gauge calibration method will produce a value for k that is close to 2 otherwise many gauges will fail to comply with their marked accuracy requirements. This can be difficult to achieve for the gauge repeatability, for instance to obtain $k$ = 2.3 requires ten repeat calibrations of the gauge. This would involve a lot of effort by the operator particularly when using a dead weight tester or pressure balance. It is not practical or warranted to make so many measurements as there may be little or no variation between the repeat measurements.

A solution to this problem has been to make three repeat measurements on the gauge [3] but use $k$ = 2 rather than the correct value for three measurements of $k$ = 4.3. It is not clear that this method produces a reliable estimate of the uncertainty at each pressure.

A new method proposed here is to carry out two complete calibration cycles on the gauge and use all the data to calculate a single value for the repeatability uncertainty. All the measurements are combined using a pooled standard deviation to estimate the uncertainty. This result has many degrees of freedom as it uses all the measurement points so $k$ will be close to 2. It does make the assumption that the gauge performance does not depend on the applied pressure. This is likely to be true for pressure gauges. We have developed this method as part of our Pressure Calibration Workshop that is held annually at MSL.

In the following sections we outline the pressure gauge calibration method and the mathematical equations used to model the gauge performance. Then we discuss the equations needed to calculate the gauge uncertainty and in particular how the uncertainty due to the gauge repeatability is found. Finally two worked examples of a gauge calibration are shown, one for a low accuracy gauge where the gauge uncertainty is much greater than that of the reference instrument and one where the gauge uncertainty is similar to the reference instrument.

## Gauge Calibration Method

For simplicity the following method describes the calibration of a pressure gauge that measures gauge pressure although the method can be applied to any type of gauge. As well it is assumed that the reference instrument (often a pressure balance, deadweight tester or high accuracy gauge) has a traceable calibration from a laboratory accredited to ISO 17025. The method is based on the requirements of EN 837 [1].

A typical pressure gauge calibration assesses the performance of the gauge at between five and ten pressure points equally spread across the scale for both rising and falling applied pressures. These pressures usually fall on the main scale markings for an analogue gauge or at an appropriate interval for a digital gauge. Zero applied pressure is a calibration point, unless the gauge has a pointer stop or scale modification that prevents the zero reading from being made.

We recommend taking the gauge around a pressure cycle from zero to full scale and back at least once before starting the calibration measurement. This cycles the gauge around a complete hysteresis loop and can show if the gauge has a major fault that would prevent calibration.

The gauge reading at each calibration point is measured for both rising and falling pressure. Rising pressure means starting at zero applied pressure (or the lowest pressure point on the gauge) and steadily increasing the pressure until the calibration pressure point is reached. Falling pressure means starting at the maximum pressure point on the gauge and then steadily decreasing the pressure to the calibration pressure point. In practice the pressure applied to the gauge is cycled from the zero (or the minimum scale point) up to the maximum scale point and then back to zero.

On reaching the desired point the pressure is held steady in readiness for taking the gauge reading. The operator must wait until both the applied pressure and gauge reading have stabilised before recording a reading. The waiting time will depend on the reference equipment, the gauge response time and size of the pressure change from the previous point. Waiting times could be as short as a few seconds for low accuracy gauges or as long as several minutes for a high resolution high pressure gauge. The system response time should be determined before starting the calibration.

The order that the gauge pressure points are applied to the gauge can be varied. One method is to calibrate the gauge sequentially by making all the rising pressure readings in increasing order until the maximum pressure is reached. Then the falling pressure readings are made in decreasing order stopping after the zero reading is taken. In this case one calibration cycles the gauge from zero to maximum and back once.

An alternative is chose a pressure point and take both the rising and falling pressure readings before continuing on to the next chosen pressure point. In this case the applied pressure will start at zero and rise steadily up to the desired pressure where a rising pressure reading is taken. Then the pressure is increased to the maximum pressure and decreased back to the pressure point where the falling pressure reading is recorded. Finally the pressure is reduced to zero.

The main advantage of this alternative order is the ease with which repeat measurements can be made, especially when using a dead weight tester or pressure balance. The repeat measurements can be made by simply taking the pressure around another calibration cycle without any need to load or unload weights. This reduces both the time and effort required to make a repeat calibration.

This alternative order has other advantages. It provides the opportunity to record a number of zero readings during the calibration which can be used to assess any drift in the gauge reading. It also allows the pressure readings to be taken in any order, rather than in a continuously increasing and decreasing sequence.

## Calibration Equation

The equation used to model the calibration is

$$p_{\mathrm{ref}} = p_{\mathrm{read}} + p_{\mathrm{corr}}$$

where $p_{\mathrm{ref}}$ is the applied reference pressure, $p_{\mathrm{read}}$ is the pressure gauge reading and $p_{\mathrm{corr}}$ is the correction that needs to be added to the gauge reading to make it equal to the reference pressure. This equation can be rearranged to find $p_{\mathrm{corr}}$: $p_{\mathrm{corr}} = p_{\mathrm{ref}} - p_{\mathrm{read}}$, which is the result needed for the pressure gauge calibration report along with the calibration uncertainty.

There are three main uncertainty components for a gauge calibration. These are: $u_{\mathrm{ref}}$ the reference standard component, $u_{\mathrm{res}}$ the gauge resolution component and $u_{\mathrm{rep}}$ the gauge repeatability. The standard uncertainty for $p_{\mathrm{corr}}$ is given by

$$u_{\mathrm{corr}} = \sqrt{u_{\mathrm{ref}}^2 + u_{\mathrm{res}}^2 + u_{\mathrm{rep}}^2}\ .$$

Here the uncertainty components are combined following section 5 of [2].

Finally the expanded uncertainty is calculated using $U_{corr} = ku_{\mathrm{corr}}$ where $k$ is the coverage factor chosen so that the uncertainty interval has a 95 percent level of confidence. The value of $k$ is calculated using the t-distribution and the effective degrees of freedom of the measurement, see Table G.2 in [2]. The effective degrees of freedom are calculated using the Welch-Satterthwaite formula found in section G.4 of [2].

The calculation of $k$ for a pressure gauge calibrated following this guide can be simplified by choosing to use $k$ = 2.2. This approximation will have a maximum error of 9 % as $k$ will be in the range 2.0 to 2.4. This error is small compared to other sources of error in the expanded uncertainty.

## Components of Uncertainty

This section discusses how the three main uncertainty components are calculated.

### *Reference Pressure Uncertainty $u_{ref}$*

This term is the standard uncertainty of the applied pressures and its value can be found in the reference instrument calibration report. A typical uncertainty statement for a dead weight tester will read:

> *'The expanded uncertainty in the pressure that is generated by the deadweight tester is 0.015 % of the calculated pressure or 0.04 kPa, whichever is greater, for pressures in the range 260 kPa to 3000 kPa.'*
>
> *The expanded uncertainty was calculated using a coverage factor $k = 2.1$ and defines an interval estimated to have a 95% level of confidence.'*

The expanded uncertainty $U_{\mathrm{ref}}$ is converted to a standard uncertainty by dividing by the coverage factor $k$ so that $u_{\mathrm{ref}} = U_{\mathrm{ref}}/k$ .

In the example above the standard uncertainty at 2000 kPa will be given by

$$u_{\mathrm{ref}}(2000\ \mathrm{kPa}) = \frac{0.015\ \%}{2.1} \times 2000\ \mathrm{kPa} = 0.14\ \mathrm{kPa}.$$

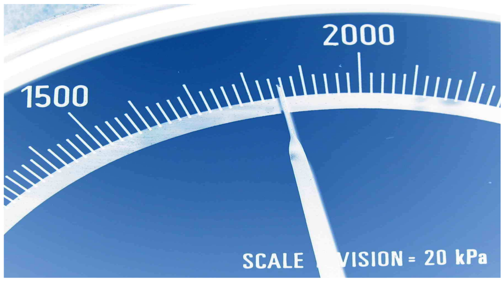
### *Resolution Uncertainty $u_{res}$*

The resolution uncertainty depends entirely on the size of the gauge resolution. The resolution is the smallest change in pressure that can be read from the gauge display or scale.

The resolution of a digital display is usually the size of the least significant displayed digit.

For the example shown above, the gauge resolution is 0.01 kPa. The resolution of some digital devices can change as the pressure increases so the resolution uncertainty will also change with pressure. Some digital displays may increment by more than one count, in this case the resolution is equal to the size of the increment.

The resolution of a gauge with an analogue scale is calculated as a fraction of the smallest scale interval. The scale interval is the smallest pressure interval marked on the gauge scale.




In the example shown above the smallest scale interval is 20 kPa. The gauge resolution is then calculated using $\text{resolution} = \text{smallest scale interval}/f$ , where $f$ is a number between 4 and 10 and represents how finely you can divide up the smallest scale interval [2]. The value of $f$ is chosen by the operator using the scale and pointer size as a guide. For most gauges it is sufficient to set $f = 4$ or 5.

The uncertainty due to the resolution is found using

$$u_{\text{res}} = \frac{\text{resolution}}{2\sqrt{3}} .$$

The $2\sqrt{3}$ factor in the denominator arises from treating the resolution error as a uniform rectangular distribution (see section 4.3.7 of [2]). For the analogue gauge example using $f = 4$, we find the resolution will be 5 kPa and $u_{\text{res}} = 5/(2\sqrt{3}) = 1.4$ kPa .

### *Repeatability Uncertainty $u_{rep}$*

The calculation of the repeatability component requires two complete calibration cycles to be made on the pressure gauge. We shall call them cycle 1 and cycle 2. The difference between the cycle 1 and 2 measurements made at each rising and falling pressure is used to calculate a pooled standard deviation that is an estimate of the gauge repeatability. Examples of this type of calculation can be found in section H.3.6 of the guide [2] and in ISO 5727-3:1994 section 8.2 [4]. This method makes the assumption that the gauge repeatability is the same at all pressures. This is likely to be true for most gauges.

The uncertainty component is calculated from

$$u_{\text{rep}} = \frac{1}{\sqrt{2}} \sqrt{\frac{\sum_{i=0}^{N} \left(p_{\text{read1},i} - p_{\text{read2},i}\right)^2}{2N}}$$

$$= \frac{1}{\sqrt{2}} \sqrt{\frac{\sum_{i=0}^{N} \left(\text{Diff}_{12,i}\right)^2}{2N}} ,$$

where $N$ is the number of rising and falling pressure points in a single calibration cycle, $p_{\text{read1,i}}$ is the i$^{\text{th}}$ gauge pressure reading from the first calibration cycle, $p_{\text{read2,i}}$ is the i$^{\text{th}}$ gauge pressure reading from the second calibration cycle. and $\text{Diff}_{12,i}$ is difference between each reading from cycle 1 and 2. The $1/\sqrt{2}$ term arises as the reported corrections are an average of the two cycles. This calculation is more straightforward than is looks, see the worked examples below.

## Worked Examples

The following examples show the details of the uncertainty calculations for the pressure gauge corrections.

### *Example 1: A Class 1 Pressure Gauge*

A class 1 gauge is a typical analogue gauge, used in industry, with a maximum permissible error of 1 % of the span. The dominant uncertainty component will usually be the gauge resolution or possibly the repeatability.

The gauge in this example has a scale from 0 kPa to 4000 kPa, with a smallest scale interval of 100 kPa and a largest permissible error of 40 kPa. The reference instrument is a deadweight tester with a calibration report stating that the expanded uncertainty is 0.01 % of the generated pressure, $k = 2.1$, across the pressure range of interest. We have chosen to calibrate this gauge at five points (including zero) evenly spaced across the scale.

The uncertainty components are as follows.

### *Reference Pressure Uncertainty $u_{ref}$*

This component is simply calculated from the uncertainty statement in the dead weight tester calibration report using

$$u_{\text{ref}}(p) = \frac{U_{\text{ref}}}{k} \times p = \frac{0.0001}{2.1} p = 0.00005p .$$

The value for $u_{\text{ref}}$ depends on $p$ the generated pressure.

Here the uncertainty of the reference instrument is very small compared to the gauge accuracy class and could be neglected. Instead we choose to keep the term but simplify the calculation by setting $u_{\text{ref}}(p) = u_{\text{ref}}(4000 \text{ kPa}) = 0.2$ kPa for all pressure values; i.e., we have set $u_{\text{ref}}$ to the value at the maximum scale pressure.

### *Resolution Uncertainty $u_{res}$*

The gauge resolution is calculated as one fifth of the smallest scale interval, i.e., $f = 5$, so

$$\text{resolution} = \frac{100\,\text{kPa}}{5} = 20\,\text{kPa} .$$

Therefore, we find that $u_{\text{res}} = 20/(2\sqrt{3}) = 5.8$ kPa .

### *Repeatability Uncertainty $u_{rep}$*

This component is calculated from the difference between the two calibration cycles. The data is shown in the table below, where Type is the pressure direction, $p_{\text{ref}}$ the reference pressure, the two sets of gauge read-

| Type | $p_{ref}$ / kPa | Cycle 1 / kPa | Cycle 2 / kPa | $Diff_{1,2}$ / kPa | $Diff_{1,2}^2$ / kPa$^2$ |
|---|---|---|---|---|---|
| Rise | 1000.2 | 1020 | 1040 | –20 | 400 |
| Rise | 2000.1 | 2040 | 2060 | –20 | 400 |
| Rise | 3000.0 | 3040 | 3060 | –20 | 400 |
| Rise | 3999.9 | 4040 | 4040 | 0 | 0 |
| Fall | 3000.0 | 3020 | 3020 | 0 | 0 |
| Fall | 2000.1 | 2000 | 2020 | –20 | 400 |
| Fall | 1000.2 | 1020 | 1020 | 0 | 0 |
| Fall | 0.0 | 20 | 20 | 0 | 0 |
| | | | | Sum Diff$^2$ | 1600 |
| | | | | *N* | 8 |
| | | | | $u_{rep}$ / kPa | 7.1 |

ings are in the Cycle 1 and 2 columns and the difference $Diff_{12,i}$ and difference squared values are shown in the last two columns.

The component $u_{rep}$ is calculated by summing up all the values of $Diff_{1,2}^2$, dividing by 2N which is two times the number of rising and falling pressure points, taking the square root of the result and multiply by $1/\sqrt{2}$ . The value obtained for this gauge is $u_{rep} = 7.1$ kPa. Often there is no difference between the readings of cycle 1 and 2 for this class of gauge. In that case $u_{rep} = 0$.

### *Standard and Expanded Uncertainty $u_c$, $U_c$*

The standard uncertainty for the gauge corrections is calculated by combining the uncertainty terms together as follows

$$u_{corr} = \sqrt{u_{ref}^2 + u_{res}^2 + u_{rep}^2}$$

$$= \sqrt{(0.2\text{ kPa})^2 + (5.8\text{ kPa})^2 + (7.1\text{ kPa})^2}$$

$$= 9.1\text{ kPa.}$$

This calculation can overestimate the uncertainty as the repeatability uncertainty may include a contribution from the resolution. In practice, keeping all the terms in the calculation does not make much difference unless $u_{res} = u_{rep}$, in which case $u_{corr}$ will be overestimated by about 40 %.

Finally the expanded uncertainty of the gauge corrections can be calculated from $U_{corr} = ku_{corr}$ , where we will set $k$ = 2.2. For this example, we find $U_{corr} = 2.2 \times 9.1 = 20.1$ kPa at an estimated 95 percent level of confidence.

### *Result*

The final result for the calibration of this gauge is a table containing the nominal pressures, the average correction for each rising and falling pressure rounded to the nearest 10 kPa and an uncertainty statement. The result section of the report could look as follows:

| $p_{nom}$/ kPa | $p_{corr}$ | |
|---|---|---|
| | Rise / kPa | Fall / kPa |
| 0 | | –20 |
| 1000 | –30 | –20 |
| 2000 | –50 | –10 |
| 3000 | –50 | –20 |
| 4000 | –40 | |

*The expanded uncertainty in each gauge correction tabulated above is 20 kPa. This expanded uncertainty was estimated by combining the uncertainties associated with the reference standards and the calibration process. The expanded uncertainty was calculated using a coverage factor $k$ = 2.2 and defines an interval estimated to have a 95 % level of confidence (see ISO Guide to the Expression of Uncertainty of Measurement, 1995).*

This gauge cannot be said to comply with the accuracy requirements for a class 1 gauge as the corrections and uncertainty are too large.

### *Example 2. A Digital Pressure Gauge*

In this example we are calibrating a 0 to 10000 kPa digital gauge, resolution 0.1 kPa, using a dead weight tester with an expanded calibration uncertainty of 0.005 %, $k = 2.0$. We have chosen to calibrate this gauge at ten pressures, including zero.

| Type | $p_{ref}$ / kPa | Cycle 1 / kPa | Cycle 2 / kPa | $Diff_{1,2}$ / kPa | $Diff_{1,2}^2$ / kPa |
|---|---|---|---|---|---|
| Rise | 999.73 | 997.5 | 997.6 | -0.10 | 0.010 |
| Rise | 1999.48 | 1994.7 | 1994.5 | 0.15 | 0.023 |
| Rise | 2999.23 | 2991.3 | 2991.4 | -0.05 | 0.002 |
| Rise | 3999.00 | 3987.8 | 3987.6 | 0.20 | 0.040 |
| Rise | 4998.76 | 4984.4 | 4984.1 | 0.25 | 0.063 |
| Rise | 5998.54 | 5979.8 | 5979.6 | 0.20 | 0.040 |
| Rise | 6998.33 | 6975.1 | 6975.1 | -0.05 | 0.003 |
| Rise | 7998.12 | 7970.1 | 7969.7 | 0.40 | 0.160 |
| Rise | 8997.92 | 8964.7 | 8964.5 | 0.20 | 0.040 |
| Rise | 9997.72 | 9958.6 | 9958.6 | -0.05 | 0.003 |
| Fall | 8997.92 | 8964.9 | 8965.0 | -0.05 | 0.003 |
| Fall | 7998.12 | 7970.9 | 7970.8 | 0.10 | 0.010 |
| Fall | 6998.33 | 6976.0 | 6975.8 | 0.20 | 0.040 |
| Fall | 5998.54 | 5981.1 | 5981.0 | 0.05 | 0.003 |
| Fall | 4998.76 | 4985.8 | 4985.5 | 0.25 | 0.063 |
| Fall | 3999.00 | 3989.3 | 3989.1 | 0.20 | 0.040 |
| Fall | 2999.23 | 2992.6 | 2992.5 | 0.10 | 0.010 |
| Fall | 1999.48 | 1995.5 | 1995.5 | 0.00 | 0.000 |
| Fall | 999.73 | 998.0 | 998.0 | 0.00 | 0.000 |
| Fall | 0.00 | 0.2 | 0.1 | 0.05 | 0.003 |
| | | | | Sum Diff$^2$ | 0.553 |
| | | | | *N* | 20 |
| | | | | $u_{rep}$ / kPa | 0.08 |

Following the previous example we have

$$u_{\mathrm{ref}}(p) = \frac{U_{\mathrm{ref}}}{k} \times p = \frac{0.00005}{2} p = 0.000025p$$

and

$$u_{\mathrm{res}} = \frac{0.1}{2\sqrt{3}} = 0.029 \text{ kPa} .$$

Again the repeatability component is calculated from the difference between the two calibration cycles and is $u_{\mathrm{rep}} = 0.08$ kPa (see the table above).

### *Standard and Expanded Uncertainty $u_c$, $U_c$*

Here the value of $u_{\mathrm{ref}}$ and $u_{\mathrm{rep}}$ are similar, so we cannot use a single value for $u_{\mathrm{ref}}$ as we did for the class 1 gauge. The gauge correction uncertainties will be pressure dependent. The table below shows the results of the uncertainty calculations.

| Nom $p$ / kPa | $u_{\mathrm{ref}}$ / kPa | $u_{\mathrm{rep}}$ / kPa | $u_{\mathrm{res}}$ / kPa | $u_c$ / kPa | $U_c$ / kPa |
|---|---|---|---|---|---|
| 0 | 0.00 | 0.08 | 0.03 | 0.09 | 0.19 |
| 1000 | 0.02 | 0.08 | 0.03 | 0.09 | 0.20 |
| 2000 | 0.05 | 0.08 | 0.03 | 0.10 | 0.22 |
| 3000 | 0.07 | 0.08 | 0.03 | 0.12 | 0.25 |
| 4000 | 0.10 | 0.08 | 0.03 | 0.13 | 0.29 |
| 5000 | 0.12 | 0.08 | 0.03 | 0.15 | 0.34 |
| 6000 | 0.15 | 0.08 | 0.03 | 0.17 | 0.38 |
| 7000 | 0.17 | 0.08 | 0.03 | 0.20 | 0.43 |
| 8000 | 0.20 | 0.08 | 0.03 | 0.22 | 0.48 |
| 9000 | 0.22 | 0.08 | 0.03 | 0.24 | 0.53 |
| 10000 | 0.25 | 0.08 | 0.03 | 0.26 | 0.58 |

The expanded uncertainty $U_c$ was calculated using $k = 2.2$. A full analysis of the effective degrees of freedom show that $k$ is between 2 and 2.1, the lower pressure uncertainties are dominated by the repeatability component while the higher pressure uncertainties are dominated by the reference pressure uncertainty.

### *Result*

The calibration result for this gauge is shown in the table below. Here the expanded uncertainty is reported in the table.

| $p_{\mathrm{Nom}}$ / kPa | $p_{\mathrm{corr}}$ | | $U_c$ / kPa |
|---|---|---|---|
| | **Rise / kPa** | **Fall / kPa** | |
| 0 | | –0.1 | 0.2 |
| 1000 | 2.2 | 1.7 | 0.2 |
| 2000 | 4.9 | 4.0 | 0.2 |
| 3000 | 7.9 | 6.7 | 0.3 |
| 4000 | 11.3 | 9.8 | 0.3 |
| 5000 | 14.5 | 13.1 | 0.3 |
| 6000 | 18.8 | 17.5 | 0.4 |
| 7000 | 23.3 | 22.4 | 0.4 |
| 8000 | 28.2 | 27.3 | 0.5 |
| 9000 | 33.4 | 33.0 | 0.5 |
| 10000 | 39.1 | | 0.6 |

The uncertainty statement for the report would read: *'These expanded uncertainties were calculated using a coverage factor k = 2.2 and define an interval estimated to have a 95 % level of confidence (see ISO Guide to the Expression of Uncertainty of Measurement, 1995).'*

## References and Bibliography


[1] Pressure gauges – Part 1: Bourdon tube pressure gauges – Dimensions, metrology, requirements and testing, EN 837-1, December 1996, CEN.

[2] Guide to the Expression of Uncertainty in Measurement, 1995, ISBN 92-67-10188-8, ISO.

[3] EA Guidelines on the Calibration of Electromechanical Manometers, EA-10/17, July 2002.

[4] Accuracy (trueness and precision) of measurement methods and results -- Part 3: Intermediate measures of the precision of a standard measurement method, Section 8, ISO 5725-3:1994.


## Further Information

If you want to know more about pressure gauge calibration, contact MSL and book in for a Pressure Calibration Workshop. See the MSL website http://msl.irl.cri.nz.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