MSL Technical Guide 10

A Simple, Narrow-Band Thermistor Thermometer

Introduction

Thermistors are ceramic semiconductor devices exhibiting a negative temperature coefficient (NTC) of electrical resistance. Although their response is non-linear, thermistors have sensitivities much greater than that of other thermometers. They are therefore suited to narrow-band applications such as precision temperature control and differential thermometry.

This technical guide describes a simple circuit that provides a voltage proportional to temperature difference. The circuit is linear within 0.1 ºC over ranges of up to ±10 ºC, capable of accuracies of a few millikelvin, and will resolve temperature differences below 50 μK.

The Circuit

The circuit diagram for the thermometer is shown in Figure 1. It consists of a thermistor, three good-quality resistors, a good-quality op-amp, and a voltage reference. The factors affecting the choice and specification of the components are discussed later.

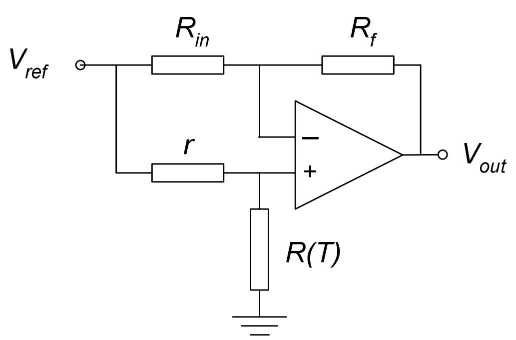

Figure 1. The simple thermistor thermometer.

The output voltage of the circuit is

$$V_{out} = \frac{R(T)R_{in} - rR_f}{R_{in}\left[r + R(T)\right]} V_{ref} \,, \qquad (1)$$

where the resistors are as indicated in the figure. The condition for zero output voltage is

$$\frac{R_{in}}{R_f} = \frac{r}{R(T_0)} \,, \qquad (2)$$

where T_0 is the nominal operating temperature of the thermometer.

One of the key features of the circuit is that the output voltage is a non-linear function of the thermistor resistance, and this non-linearity can be used to compensate for the thermistor non-linearity. The thermistor has a resistance-temperature relationship approximated by

$$R(T) = R(T_0)\exp\left(\frac{\beta}{T} - \frac{\beta}{T_0}\right) , \qquad (3)$$

where T is the temperature in kelvin. Typical values for the β coefficient, which are characteristic of the thermistor material, are in the range 2000 K to 6000 K.

The thermometer response can be characterised in terms of its sensitivity (the rate of change of output with temperature) and its quadratic and cubic non-linearity. It is found that the sensitivity is maximised and the quadratic non-linearity eliminated if

$$r = \frac{\beta - 2T_0}{\beta + 2T_0} R(T_0) \,. \qquad (4)$$

With the linearising resistor, r, set to this value and the gain setting resistors R_{in} and R_f set to the ratio given by Equation (2), the output of the thermometer is approximately

$$V_{out}(T_0 + \Delta T) \approx -\frac{\beta + 2T_0}{2T_0^2}\left[\Delta T - \frac{1}{12}\frac{\beta^2}{T_0^4}\Delta T^3\right] V_{ref} \,. \qquad (5)$$

Thus, the thermometer has a small residual cubic non-linearity (a term proportional to ΔT^3).

An example

Thermistors come in a very wide range of resistances with the R(298.15 K) values ranging from 100 Ω to 1 MΩ. The thermistor resistance should be chosen to minimise the various error effects (see below), which may increase or decrease with thermistor resistance. As a general guide, thermistors with an operating resistance in the range 1 kΩ to 50 kΩ are typical.

Here we consider a thermometer operating over the range 10 ºC to 40 ºC using a thermistor with the $R(t)$ relationship given in Table 1. (Note that a lower case t is used for temperatures in degrees Celsius.)

Step 1: Determine the β Value

The β value for the thermistor can be calculated using Equation (3) and any pair of $R(T)$ values. The solver function found in most spreadsheet applications can also be used to fit Equation (3) to the table values. In this case, T_0 was set to 298.15 K (25 ºC) and R(298.15 K) was set to 10 000 Ω. The fitted β value is 3890 K.

Table 1: Thermistor resistance vs temperature.

t /ºC	*R* /Ω	*t* /ºC	*R* /Ω	*t* /ºC	*R* /Ω
10	19900	20	12500	30	8060
11	18970	21	11940	31	7722
12	18090	22	11420	32	7402
13	17260	23	10920	33	7100
14	16470	24	10450	34	6807
15	15710	**25**	**10000**	35	6532
16	15000	26	9574	36	6270
17	14330	27	9165	37	6017
18	13680	28	8779	38	5777
19	13070	29	8410	39	5546
20	12500	30	8060	40	5329

Step 2: Set the Resistor Values

From Equation (4), the linearising resistor is 7342 Ω. To keep the balance condition simple and minimise amplifier bias-current effects, we also choose *Rin* = 7342 Ω, and R_f = 10 000 Ω.

Step 3: Set the Reference Voltage

The choice of reference voltage is influenced by the accuracy required and the ability of the thermistor to disperse the heat generated in thermistor by the sensing current. The maximum dissipation occurs when $R(T) = r$, and is given by $P_{\max} = V_{ref}^2 / 4r$. For the example thermistor and V_{ref} = 1 V, the maximum dissipation is 34 μW. For this thermistor, the dissipation constant in stirred oil is 8 mW/ ºC, which is equivalent to a thermal resistance of 125 ºC/W. Hence, the maximum temperature rise of the thermistor is 4.25 mK.

Step 4: Determine the Approximate Sensitivity

Figure 2 plots the output voltage as a function of the thermistor temperature. A good estimate of the temperature can be calculated from the measured voltage with the equation

$$T_{meas} = T_0 - \left(\frac{2T_0^2}{\beta + 2T_0}\right)\left(1 - \frac{\beta^2 \Delta T_z^2}{12 T_0^4}\right)\frac{V_{out}}{V_{ref}} . \qquad (6)$$

The constant ΔT_z provides a small correction to minimise the cubic non-linearity and is about 0.39 times the temperature range. With V_{ref} = 1 V, the sensitivity is about –25.2 mV/ºC. This sensitivity is about 500 times that of thermocouples and 50 times that of a platinum resistance thermometer, and highlights the very high sensitivity obtainable with thermistors.

Step 5: Determine the Non-Linearity

The term including ΔT_z in the second parenthesis of Equation (6) forces the cubic non-linearity to be zero at three temperatures, $T_0, T_1 = T_0 + \Delta T_z$, and $T_2 = T_0 - \Delta T_z$. This reduces the maximum cubic error by about 2.5 times. The residual cubic error is then

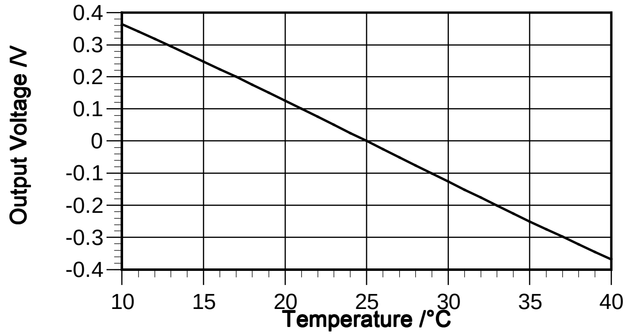

Figure 2. The circuit output vs temperature.

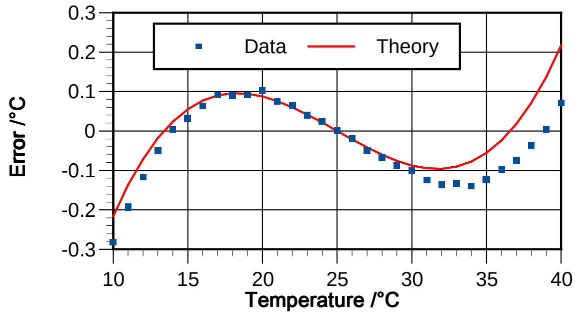

Figure 3. Non-linearity of the circuit. The points show the error in the measured temperature as calculated from the data of Table 1, Equation (1), and Equation (6). The line is Equation (7) with ΔTz = 11.6 ºC.

$$T - T_{meas} = -\frac{1}{12}\frac{\beta^2}{T_0^4}(T - T_0)(T - T_1)(T - T_2) . \qquad (7)$$

Figure 3 compares the non-linearity as calculated from theory and from the data of Table 1.

Step 6: Calibration

If resistors with a 1% tolerance are used in the circuit, the temperature can be measured with uncertainties of the order of 0.5 ºC simply by using Equation (6). The accuracy in practice should be confirmed by checking the thermometer against a reference thermometer at a few points over the operating range.

For the best uncertainties, the thermometer should be calibrated in a stirred bath against a standard platinum resistance thermometer. In this case uncertainties of a few millikelvin can be achieved. The temperature-voltage relationship should be sampled every two degrees or so and fitted to the equation

$$T_{meas} = A + BV_{out} + CV_{out}^2 + DV_{out}^3 . \qquad (8)$$

This equation will correct for any residual quadratic or cubic non-linearity.

Sources of Uncertainty

The following sub-sections describe the various sources of uncertainty associated with the implementation and use of the thermometer.

Thermistor Manufacturing Tolerance

Good quality thermistors made for temperature measurement are normally manufactured with a tolerance of either 0.1 ºC or 0.2 ºC.

Temperature Range – Non-Linearity

According to Equation (7) the non-linearity of the circuit increases as the cube of the temperature range. Therefore doubling the temperature range from 30 ºC to 60 ºC will increase the peak non-linearity to about 0.8 ºC. Additionally, there are high-order effects, just apparent in Figure 3, that become more significant with wider ranges. For wider temperature ranges, more sophisticated linearisation schemes may be required.

Thermistor Stability

The most stable thermistors are glass-bead types, followed by epoxy-bead thermistors. If restricted to temperatures below 90 ºC, they exhibit drifts of no more than a few millikelvin per year. Some selected types are available with stabilities better than 1 mK per year. In general the wider and more frequent the temperature excursions, the poorer the long term stability.

Self Heating

The self-heating in thermistors is proportional to the power dissipated and given by

$$\Delta T(I) = (\rho_{\mathrm{th}} + \rho_{\mathrm{env}})\, I^2 R(T)\,, \tag{9}$$

where I is the sensing current, ρ_{env} is the thermal resistance of the media surrounding the thermistor, and ρ_{th} is the thermal resistance of the thermistor (equal to the reciprocal of the dissipation constant).

For the circuit described here, the sensing current is given by $I = V_{ref} / \left(r + R(T)\right)$, and this leads to a self heating of

$$\Delta T(I) = (\rho_{th} + \rho_{env}) \frac{V_{ref}^2 R(T)}{\left[r + R(T)\right]^2}\,, \tag{10}$$

which is a maximum when $R(T) = r$. Hence,

$$\Delta T(I)_{\max} = (\rho_{th} + \rho_{env}) \frac{V_{ref}^2}{4r}\,. \tag{11}$$

If the self-heating error is constant, then it can be corrected by calibration. More typically, the self heating varies with turbulence in the environment, especially when measuring the temperatures of fluids such as air or water.

If the thermistor is used for precision temperature control applications requiring a resolution below the self heating error, then there are additional requirements on the stability of the reference voltage and the thermal resistance.

Lead Resistance

The circuit in Figure 1 makes a two-wire measurement of the thermistor resistance, and this necessarily includes the resistance of the lead wires to the thermistor. The effect on the temperature measurement is approximately

$$\Delta T_L = -\frac{T^2}{\beta} \frac{R_L}{R(T)}\,, \tag{12}$$

where R_L is the lead resistance. For the example circuit, the error is about 2.3 mK per ohm of lead resistance. If long leads are essential, a more complicated four-wire measurement is required.

Insulation Resistance

At low temperatures, the thermistor resistance becomes very large, often greater than 10 MΩ. In such applications, care should be taken to ensure that the electrical insulation on the lead wires does not shunt the measuring current. The error due to an insulation resistance R_{ins} is

$$\Delta T(R_{\mathrm{ins}}) = \frac{T^2}{\beta} \frac{R(T)}{R_{\mathrm{ins}}}\,, \tag{13}$$

For the example, an insulation resistance of 100 MΩ at 25 ºC causes an error of 2.3 mK.

Resistor Tolerances

The sensitivity of the circuit to errors in the resistor values can be evaluated as

$$\Delta T(R_i) = -\left(\frac{R_i}{V_{ref}} \frac{dV_{out}}{dR_i}\right)\left(\frac{2T_0^2}{\beta + 2T_0}\right)\frac{\Delta R_i}{R_i}\,, \tag{14}$$

where R_i is one of R_{in}, R_{f}, or r. This describes the temperature error for a relative change in the respective resistor. Typically $r/R(T)$ is about 0.71, and Equation (8) can be approximated by

$$\left|\Delta T(R_i)\right| = \left(\frac{T_0^2}{\beta + 2T_0}\right)\frac{\left|\Delta R_i\right|}{R_i}\,. \tag{15}$$

This corresponds to about 0.2 ºC for each 1% error or change in the resistor values. Thus, if three resistors with a 1% uncertainty in their resistance values are used in the circuit, they will contribute √3 × 0.2 ºC = 0.34 ºC uncertainty to the temperature measurement.

Resistor Temperature Coefficients

The effect of the manufacturing tolerances in the resistors can be removed by calibration. However the resistors may drift with time or ambient temperature. Common metal-film resistors have temperature coefficients of about 0.005%/ºC. Since this affects all three resistors, a 1 ºC change in ambient temperature may cause as much as 3 mK change in the thermometer output.

Reference Voltage Stability

In addition to self-heating effects, the reference voltage also affects the output signal. If the balance condition (Equation (2)) is satisfied the output voltage is zero independent of the reference voltage. As the temperature departs from T_0, the sensitivity of the circuit to changes in the reference voltage increases. Hence,

$$\Delta T(V_{ref}) = (T_0 - T)\frac{\Delta V_{ref}}{V_{ref}} \quad (16)$$

Thus, a 1% error or change in the reference voltage will lead to a 0.01 ºC error for each 1 ºC deviation from the nominal temperature.

Amplifier Offset Voltage

All amplifiers introduce a voltage error usually represented by an equivalent input offset voltage, V_{os}. The resulting temperature error is

$$\Delta T(V_{os}) = \frac{2T_0^2}{\beta - 2T_0}\frac{V_{os}}{V_{ref}}. \quad (17)$$

For the example circuit, an op-amp with a 25 µV offset voltage will introduce about 1.4 mK error.

Amplifier Bias Currents

The input bias currents of amplifiers have a very similar effect to the offset voltage:

$$\Delta T(I_b) = \frac{1}{V_{ref}}\left(\frac{2T_0^2}{\beta - 2T_0}\right) \times \left(I_b^+ \frac{rR(T)}{r + R(T)} - I_b^- \frac{R_{in}R_f}{R_{in} + R_f}\right), \quad (18)$$

where I_b^- and I_b^+ are the bias currents for the inverting and non-inverting inputs respectively. Each bias current 'sees' an impedance equal to the parallel combination of the resistors connected to the respective input. If this impedance is less than V_{os}/I_b, the contribution of the bias currents will be less than that caused by the offset voltage. In good quality op-amps, the bias currents are about 1 nA, and the offset voltage is about 20 µV, so the effect is negligible if the resistances are less than 20 kΩ.

For most op-amps the two bias currents are similar so the effect is minimised by choosing $R_{in} = r$, and $R_f = R(T)$.

Calibration Uncertainties

The uncertainties in measured temperature introduced through the calibration of the thermistor depend on the standards held by the calibration laboratory.

The ITS-90 temperature scale is currently defined in terms of the resistance of standard platinum resistance thermometers (SPRT). The best accuracy of SPRTs normally varies between about 0.2 mK near 0 ºC to 2 mK near 100 ºC.

In the process of calibration, the thermistor and the SPRT must be immersed in the same environment, usually a stirred water or oil bath. The stability and uniformity of the best commercial calibration baths is about 0.5 mK.

References

W R Beakley, "The design of thermistor thermometers with linear calibration", *J. Sci. Instrum.*, **28**, 176–179, 1951.

C D Vaughn, J Gartenhaus, and G F Strouse, "NIST Calibration uncertainties of Thermistor Thermometers over the range from –50 ºC to 90 ºC". *Proc. 2005 NCSL Workshop*.

Prepared by D R White, December 2005.