MSL Technical Guide 31

Using calibrated RF power sensors

Introduction

Power is a fundamental quantity in radio and microwave frequency systems. At lower frequencies, voltage and current are often measured, but at high frequencies these behave in complicated ways that makes their measurement difficult.

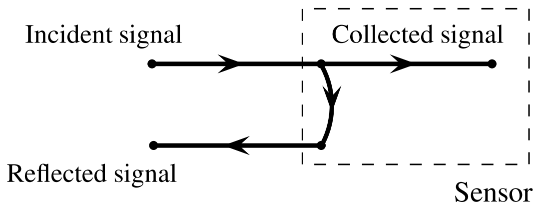

Figure 1: Signal flow in a high-frequency measurement

When measuring a high-frequency signal, some incident power is reflected away from the sensor and not detected (Figure 1). The remaining signal is collected, but not all of it is registered by the sensing element, because there is also dissipation within the sensor body.

These effects mean that the power registered by a sensor is not actually the incident signal power, which was intended to be measured. However, by calibrating a device, these measurement errors can be corrected.

Measurement principles

Effective efficiency

The characteristic of a power sensor that can be linked to DC quantities, making power measurements traceable to the SI, is called the *effective efficiency*

$$\eta_\mathrm{e} = \frac{\text{detected power}}{\text{collected power}} \ .$$

This is the ratio of power dissipated in the sensing element to total power dissipated within the sensor. A sensor that detected all the collected power would have 100% effective efficiency.

Calibration factor

A more practical sensor characteristic is the *calibration factor*

$$K = \frac{\text{indicated power}}{\text{incident power}} \ ,$$

which can be used to convert the power registered by a sensor to an estimate of the incident power.

The calibration factor combines information about the effective efficiency with information about the reflected signal. It can be expressed as

$$K = \eta_\mathrm{e} \left(1 - |\boldsymbol{\Gamma}_\mathrm{s}|^2\right) \ ,$$

where $|\boldsymbol{\Gamma}_\mathrm{s}|$ is the magnitude of the (complex-valued) reflection coefficient of the power sensor. The calibration factor depends on frequency, because both η_e and $\boldsymbol{\Gamma}_\mathrm{s}$ are frequency dependent.

Calibration

When a sensor is calibrated, either the effective efficiency or the calibration factor can be reported; although usually it is the calibration factor, which is more useful in many cases.

The sensor reflection coefficient will also be reported (often, only the magnitude). This can be used to convert between effective efficiency and calibration factor. More importantly, however, it is also needed to assess the error due to mismatch in any measurement set-up.

Mismatch

Any signal reflected away from a power sensor will impinge on other elements of a network and be reflected again. As signal bounces back and forth between network discontinuities, the power level seen at the sensor will be affected by interference effects.

When a signal generator and power sensor are connected, the incident power at the sensor is

$$P_\mathrm{inc} = P_\mathrm{g0} \frac{1}{|1 - \boldsymbol{\Gamma}_\mathrm{g}\boldsymbol{\Gamma}_\mathrm{s}|^2} \ ,$$

where P_{g_0} is the generator power and $\boldsymbol{\Gamma}_g$ is the generator reflection coefficient.

The ratio of 'power delivered' to 'power available' (i.e., P_{inc}/P_{g_0}) may be expressed as a mismatch term

$$M = |1 - \boldsymbol{\Gamma}_g \boldsymbol{\Gamma}_s|^2 \ ,$$

which is sometimes called the *mismatch loss* (when reported in dB).

Putting it together

When K, the calibration factor, and M are known, the generator power (usually the quantity of interest) is related to the power sensor response P_{sen} by

$$P_{g_0} = P_{sen} \frac{M}{K} \ .$$

However, the sensor response is not available to the user directly. Instrumentation processes the response before rendering an indication of power.

Power meters

Modern systems aim to simplify measurements by automating steps wherever possible. Nevertheless, when evaluating the overall measurement uncertainty, it is important to understand the metering system because, automated or not, any step that incurs measurement error must be considered.

It is helpful to think of a power meter as an instrument that converts a sensor response into a 'correct' reading, or indication. To do so, an offset must be adjusted, so that the meter reads zero when no power is applied, and the meter gain must be adjusted, so that a reference power level is correctly indicated (typically 1 mW).

We may express the relationship between the power meter reading p_m and the sensor response as

$$p_m = B P_{sen} + A \ ,$$

where A is the meter offset and B is the meter gain.

Most power meters rely on the user to perform a zero adjustment at appropriate times. Some also allow a user to perform the gain adjustment by connecting the sensor to a reference source, while others rely on the stability of factory settings in the instrumentation.

Modern power meters (and 'smart sensors' which effectively package metering instrumentation with the sensor) can store a calibration factor and apply the correction above automatically, so that an estimate of incident power is indicated directly.[1]

The instrumentation will contribute some noise to a result and, if the zero setting has been performed some time before the measurement, drift may also contribute to the meter indication. We may represent these effects by adding terms to the equation above

$$p_m = B P_{sen} + A + D + N \ ,$$

where D is the drift in the zero setting and N is an error due to system noise.

Measurement error and uncertainty

A measurement only provides an estimate of the power level of interest; the exact value cannot be known. The difference between the estimate and the true value is important, so we evaluate the typical size of this difference. This is called evaluating the measurement uncertainty.

Measurement model

Every uncertainty analysis is based on a model of the measurement system [1]. That is, an equation that relates the quantity intended to be measured to all other quantities that influence a measurement result. The model can be more or less detailed, depending on the accuracy required.

We can develop a measurement model by starting with the equation for generator power in terms of the sensor response, the mismatch and the calibration factor

$$P_{g_0} = P_{sen} \frac{M}{K} \ .$$

We then combine this with the meter model, including gain and offset adjustments, noise and drift

$$\begin{aligned} P_{g_0} &= \left(\frac{p_m - A - D - N}{B} \right) \frac{M}{K} \\ &= \frac{p_m}{B} \left(1 - \frac{A + D + N}{p_m} \right) \frac{M}{K} \ . \end{aligned}$$

Now, the meter performance will be good, so $A \approx 0$ and $B \approx 1$ are reasonable estimates; $M \approx 1$ is usually assumed too, unless the complex reflection coefficients $\boldsymbol{\Gamma}_g$ and $\boldsymbol{\Gamma}_s$ have been measured, and finally $D \approx 0$ and $N \approx 0$. So we obtain the familiar estimate

$$P_{g_0} \approx \frac{p_m}{K} \ .$$

[1] These systems are not smart enough, however, to determine the measurement frequency, which must be supplied by the user.

However, to work out the uncertainty in this result, as an estimate of the actual generator power, we need to consider the uncertainty of each of the other estimates made here ($A \approx 0$, etc).

Measurement uncertainty

Every term in the measurement model except $p_{\rm m}$ is uncertain; we have only estimates of these quantities, each with an associated error.[2] To work out the uncertainty in our estimate of $P_{\rm g0}$ we must combine all the components of uncertainty.

There are two simple rules for combining uncertainties when the associated errors are independent [2]. The first rule applies when terms are added or subtracted in the measurement model. In that case, the associated uncertainties should be squared and added together. Hence the combined uncertainty of $A + D + N$ is[3]

$$u(A + D + N)^2 = u(A)^2 + u(D)^2 + u(N)^2 \ .$$

The second rule applies when terms are multiplied or divided in the measurement model. In that case, the associated *relative* uncertainties should be squared and added together. We apply this rule to obtain the relative uncertainty of the generator power

$$\left(\frac{u(P_{\rm g0})}{P_{\rm g0}}\right)^2 = \left(\frac{u(B)}{B}\right)^2 + \left(\frac{u(A+D+N)}{p_{\rm m}}\right)^2 + \left(\frac{u(M)}{M}\right)^2 + \left(\frac{u(K)}{K}\right)^2 \ .$$

Uncertainty components

Calibration factor

Calibration factor values are typically a little less than unity. The associated relative measurement uncertainties can vary between a few tenths of a percent to a few percent, depending on the frequency and the accuracy of the method used to calibrate the sensor.

Mismatch

Mismatch depends on the characteristics of the sensor and the network in which the sensor is being used, so the user must evaluate this uncertainty component; it cannot be done by the calibration laboratory.

Usually the phase of $\boldsymbol{\Gamma}_{\rm g}\boldsymbol{\Gamma}_{\rm s}$ is unknown, which means that

$$M = |1 - \boldsymbol{\Gamma}_{\rm g}\boldsymbol{\Gamma}_{\rm s}|^2$$

could be greater or less than unity. So we take $M \approx 1$ and evaluate a *mismatch uncertainty*, which depends on $|\boldsymbol{\Gamma}_{\rm g}|$ and $|\boldsymbol{\Gamma}_{\rm s}|$,

$$u(M) = \sqrt{2}\,|\boldsymbol{\Gamma}_{\rm g}||\boldsymbol{\Gamma}_{\rm s}| \ .$$

Noise, zero setting and drift

The manufacturer will generally supply information about noise, zero-offset error and the stability of the zero setting over time. Noise will usually be affected by sensor configuration settings, such as the number of samples that are averaged internally.

Rather than rely on the specifications, a user may prefer to collect a sample of power readings and calculate the sample mean and standard deviation.[4] The sample mean can be used instead of the single reading $p_{\rm m}$, but there are two cases to consider for the uncertainty component associated with noise.

If the sample of data is collected quickly compared to the sensor drift time scale then the *sample standard deviation of the mean* can be associated with the uncertainty component for noise

$$u(N) = \frac{\text{sample standard deviation}}{\sqrt{\text{no. readings}}} \ .$$

On the other hand, if drift during the measurements is likely, the sample standard deviation should be associated with the combined uncertainty due to noise and drift

$$u(N + D) = \text{sample standard deviation} \ .$$

Gain

When the power sensor gain is not adjustable by the user this term may be ignored. In effect, the sensor calibration factor incorporates the gain factor.

Some manufacturers supply information about a power reference provided for sensor adjustment. In that case

$$B = \frac{p_{\rm nom}}{P_{\rm ref}} \frac{M_{\rm c}}{K_{\rm c}} \left(1 - \frac{A}{p_{\rm nom}}\right) \ ,$$

where $p_{\rm nom}$ is the nominal value of the reference and $P_{\rm ref}$ is the actual reference value. The subscript ‘c’ indicates

[2] Here, we assume that actual meter reading is stable, so $p_{\rm m}$ is exactly known because we can see it, or read it with a computer. The situation where the meter reading varies due to noise is covered in the section on Uncertainty Components, below.

[3] We use the lower case ‘u’, as in $u(A)$, to represent the *standard uncertainty* of an estimate [1].

[4] In the *Guide to the Expression of Uncertainty* this is called a type-A evaluation of uncertainty [1].

the mismatch and calibration factor during the gain setting. Using the product-quotient rule again, the relative uncertainty

$$\left(\frac{u(B)}{B}\right)^2 = \left(\frac{u(P_{\mathrm{ref}})}{P_{\mathrm{ref}}}\right)^2 + \left(\frac{u(M_{\mathrm{c}})}{M_{\mathrm{c}}}\right)^2 + \left(\frac{u(K_{\mathrm{c}})}{K_{\mathrm{c}}}\right)^2 + \left(\frac{u(A)}{p_{\mathrm{nom}}}\right)^2 .$$

This expression for $u(B)/B$ can be used to get a more detailed equation for $u(P_{\mathrm{g0}})/P_{\mathrm{g0}}$ when the user sets the meter gain[5]

$$\left(\frac{u(P_{\mathrm{g0}})}{P_{\mathrm{g0}}}\right)^2 = \left(\frac{u(P_{\mathrm{ref}})}{P_{\mathrm{ref}}}\right)^2 + \left(\frac{u(M_{\mathrm{c}})}{M_{\mathrm{c}}}\right)^2 + \left(\frac{u(K_{\mathrm{c}})}{K_{\mathrm{c}}}\right)^2 + \left(\frac{u(M)}{M}\right)^2 + \left(\frac{u(K)}{K}\right)^2 + \left(\frac{u(D+N)}{p_{\mathrm{m}}}\right)^2 + \left(\frac{u(A)}{p_{\mathrm{m}}} - \frac{u(A)}{p_{\mathrm{nom}}}\right)^2 .$$

Other uncertainty components

Other sources of error are sometimes considered in an uncertainty calculation, such as: linearity, finite display resolution and connector repeatability. Any of these can be included as additional relative uncertainty terms in the equation for $u(P_{\mathrm{g0}})/P_{\mathrm{g0}}$.

Finite resolution

When the meter indication has a fixed number of digits, the component of relative uncertainty associated with truncating the indication is

$$\frac{1}{\sqrt{12}}\frac{\delta p}{p_{\mathrm{m}}} ,$$

where δp is the power represented by one least significant digit.

Connector repeatability

Connector performance depends on many factors, so it is best to evaluate this component of uncertainty by measurements. The power should be recorded when the sensor is reconnected a number times (4 – 6), changing the orientation of the connectors each time.

A relative uncertainty component for connector repeatability can be obtained from the sample of repeated power measurements by calculating

$$\frac{\text{sample standard deviation}}{\text{sample mean}} .$$

Note that the measurements will also be affected by instrument noise. So some judgement will be required as to whether the connector repeatability is significant compared to the noise in the measurement system.

Engineering units

The equations in this technical guide are correct when used with quantities expressed in SI units (sometimes called 'linear units' by the RF community).[6] However, manufacturer data is often reported in *ad hoc* engineering units, in which case conversions must be applied.

Power uncertainty in decibels

The uncertainty of a power value y_{dB}, expressed in decibels (dB), can be converted to a relative uncertainty in linear units as follows

$$\frac{u(y_{\mathrm{lin}})}{y} = 10^{u(y_{\mathrm{dB}})/10} - 1 ,$$

provided $u(y_{\mathrm{lin}})/y \ll 1$.

Reflection coefficient as a return loss

When the magnitude of a reflection coefficient is reported in units of return loss the following transformation can be used

$$\rho = |\boldsymbol{\Gamma}| = 10^{-y_{\mathrm{RL}}/20} .$$

Reflection coefficient as a standing wave ratio

When the magnitude of a reflection coefficient is reported as a standing wave ratio, or voltage standing wave ratio (SWR, or VSWR), the following transformation can be used

$$\rho = |\boldsymbol{\Gamma}| = \frac{r-1}{r+1} ,$$

where r is the SWR.

Example

We will evaluate the measurement uncertainty for a 100 μW signal. The sensor is a fixed-gain type, which has been calibrated. From the calibration certificate, the

[5]The last term in this equation takes account of correlation associated with the zero-setting error.

[6]These 'linear' units are dimensionless ratios of SI quantities. The 'linear' classification distinguishes them from other popular engineering units that are related by non-linear conversion functions.

sensor specifications document and the signal generator specifications document we obtain the following data.

component	value	unit
Generator port $\lvert\Gamma\rvert$	< 1.8	VSWR
Sensor $\lvert\Gamma\rvert$	0.083	(linear)
$u(K)$	0.8	$\%$
Zero set	< 50	nW
Zero drift	< 20	nW/h
Noise	< 30	nW

In addition, we note that, from manufacturer documentation, the sensor non-linearity error is negligible at this signal level.

A series of repeat measurements were made to assess the connector repeatability. These results yield a relative uncertainty component of $0.16\ \%$.

Interpreting manufacturer specifications

There can be no hard and fast rule on how to interpret manufacturer-supplied specifications when preparing an uncertainty calculation. Manufacturers tend to adopt their own interpretations for terms like 'specification' and 'typical' value, for instance. Nevertheless, the most important terms in the uncertainty budget should be independently verified if possible.

In the tabulated data above, we will assume the upper limits on the zero set, zero drift and noise specification values to be about twice the standard uncertainties required for the uncertainty calculation.

The generator SWR, on the other hand, will be taken at face value, because we are interested in $|\Gamma|$ in this case, not $u(|\Gamma|)$.

Calculating the uncertainty

Zero set, drift and noise

From the table data, and dividing each value by two,

$$u(A+D+Z)^2 = (25 \times E^{-9})^2 + (10 \times E^{-9})^2 + (15 \times E^{-9})^2 = 950 \times 10^{-18}$$

from which we obtain

$$\left(\frac{u(A+D+Z)}{p_\mathrm{m}}\right)^2 = 95 \times 10^{-9}\ .$$

Mismatch

We must convert the generator port SWR to linear units

$$1.8\ \text{SWR} \rightarrow 0.286\ \text{(lin)}$$

then evaluate

$$u(M) = \sqrt{2} \times 0.286 \times 0.083 = 3.36 \times 10^{-2}\ .$$

We must square this number before using it in the combined uncertainty calculation.

The combined uncertainty

The calibration factor and connector repeatability are already expressed as relative uncertainties, so we may now combine the different components

$$\left(\frac{u(P_{\mathrm{g0}})}{P_{\mathrm{g0}}}\right)^2 = (950 \times 10^{-18}) + (3.36 \times 10^{-2})^2 + (0.008)^2 + (0.0016)^2 = 1.20 \times 10^{-3}\ .$$

Taking the square root we obtain

$$\frac{u(P_{\mathrm{g0}})}{P_{\mathrm{g0}}} = 3.5 \times 10^{-2}$$

which is a relative uncertainty of $3.5\ \%$, or an absolute uncertainty of $3.5\ \mu$W on a $100\ \mu$W signal.

Clearly the contribution from mismatch, which is four times the size of the calibration factor uncertainty, dominates the combined measurement uncertainty for power in this measurement. This is unfortunate, because the mismatch uncertainty calculation uses a manufacturer's specification for the generator port $|\boldsymbol{\Gamma}_\mathrm{g}|$ which will tend to be conservative. It is, however, difficult to make an independent measurement of $|\boldsymbol{\Gamma}_\mathrm{g}|$, so in practice the mismatch uncertainty can usually only be improved by ensuring that the sensor $|\boldsymbol{\Gamma}_\mathrm{s}|$ is as small as possible (because $u(M) = \sqrt{2}\,|\boldsymbol{\Gamma}_\mathrm{g}||\boldsymbol{\Gamma}_\mathrm{s}|$).

Prepared by Blair Hall, October 2014.

References

[1] BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, and OIML. *Evaluation of measurement data - Guide to the expression of uncertainty in measurement JCGM 100:2008 (GUM 1995 with minor corrections).* BIPM Joint Committee for Guides in Metrology, Paris, Sèvres, 1 edition, 2008. [on-line: JCGM 100:2008]

[2] www.uncertainnumbers.com/notes

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