MSL Technical Guide 9

Uncertainty Calculations for Remote Clocks

Introduction

The "Line 6" common view time transfer technique allows the frequencies of geographically separated precision oscillators to be compared. If one of the oscillators involved has a traceable calibration, a traceable calibration of the remote oscillator can be made. However for a traceable measurement an estimate of the uncertainty is needed in addition to the measured values.

This guide describes the uncertainty calibrations made for the measurements reported in the MSL bulletins and suggests how users of the bulletins can calculate an uncertainty for the frequency of their remote oscillator.

The calculations assume that the measurements are being used for a frequency calibration of the remote oscillator.

The Calculation of Uncertainties

The calculation of uncertainties in time and frequency calibrations is not well covered in the “Guide to the Calculation of Uncertainties” (GUM) [1]. The problem with the GUM recommendations on the calculation of uncertainty is illustrated in the following example. The figure below shows the results of hourly time interval measurements of MSL’s oldest caesium clock versus the one pulse per second (pps) signal that is UTC(MSL). In total, the figure has some 1000 points and a simple way to determine the uncertainty in this calibration would be to calculate a simple standard deviation of the results as described in the GUM which gives and expanded uncertainty of 226 ns. This is clearly too large as it includes the constant frequency difference between the clocks. Dividing the result by √N gives a standard uncertainty of 4.8 ns. Figure 2 shows the residuals left after subtracting

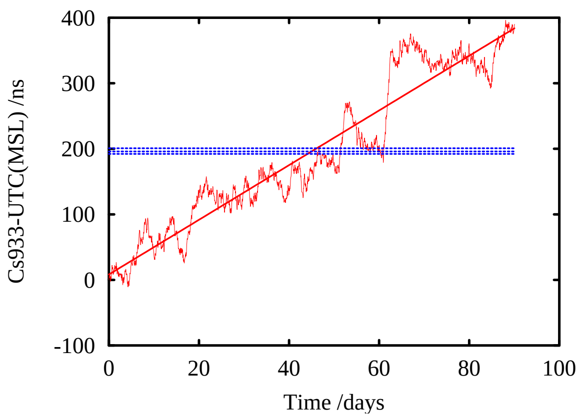


Figure 1. Hourly time interval measurements between an MSL caesium clock and UTC(MSL).

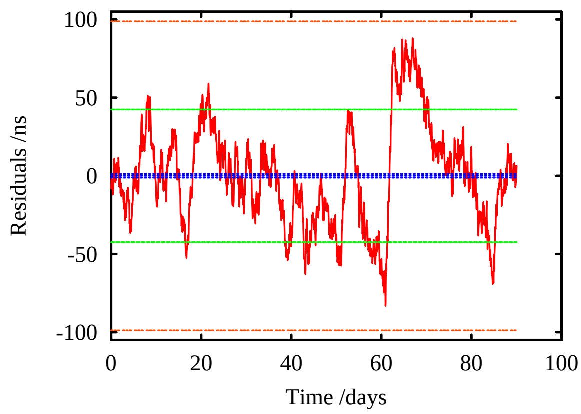


Figure 2. The residuals from the data of figure 1 after ~4ns/day has been removed. The blue lines show the uncertainty for a simple standard deviation of the result, the red lines a maximum/minimum calculation and the green line a TDev calculation.

off the linear drift. The dotted blue lines shown in the figure are 4.8 ns uncertainty. This clearly does not include the expected 95% of the measurements.

The other method, recommended by the GUM in section 4.3.7 is to simply take of the upper and lower limits of the residuals and treat these as a rectangular distribution. The expanded uncertainty (U) for a coverage factor of 2 is then given by:

$$U = 2 \times \sqrt{(\max - \min)/3}\ ,$$

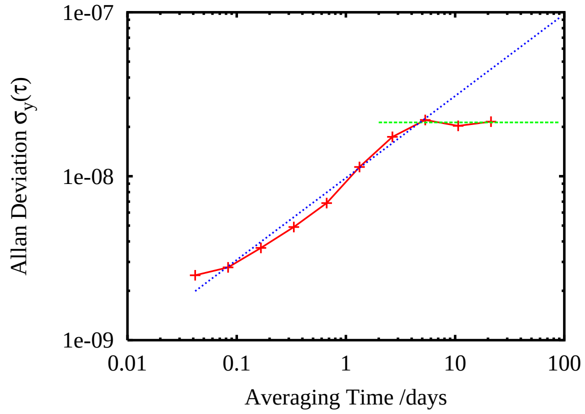
where max and min are the upper and lower limits respectively. The expanded uncertainty calculated using this method (assuming $k = 2$) is shown in the figure above with the red dashed lines. This is clearly an over estimate but is easy to calculate.

The best way to determine the uncertainty is to calculate the Allan deviation of the measurements. Note that this method is recommended by the GUM in section 4.2.7. See reference [2] for a detailed discussion of the Allan deviation.

For measurements of time stability the time variance (TDev), which is a modified version of the Allan Variance, should be used.

A useful program to calculate these quantities is called “AlaVar”. IT can be downloaded from the Internet and used for free [2]. The documentation with the program includes definitions for the different types of variances.

The figure below shows the TDev $\sigma_x(\tau)$ for the raw data used to generate the previous figure. For the 90 days of data, the $\sigma_x(\tau)$ is ~20 ns giving an expanded uncertainty of ~40 ns. This is shown in the previous figure as the green lines. It can be seen that these lines give the best estimate of the uncertainty as they correctly appear to encompass around 95% of the points in the figure.



The advantage of the Allan variance methods are that:

- they give a smaller and more accurate measure of the uncertainty;
- the uncertainty at different averaging times can be determined;
- although it is beyond the scope of this guide, the slopes of the lines in the graph can provide information about the types of noise in the measurements.

The Line 6 Technique

At 2:35 pm (New Zealand Standard Time), MSL makes a time interval measurement started with a pulse derived from the synchronisation pulse of the sixth line of the television picture and UTC(MSL). The values measured at MSL are then reported in the MSL weekly and monthly bulletins. Similar time interval measurements between a line six pulse from a TV for a pps signal from the clock are made at the remote site.

The MSL Uncertainties

There are four uncertainty components which contribute to the uncertainty in the time interval measurements of the television Line 6 signal made at MSL. These components are described in the following sections.

Comparison Noise

There is some jitter in the time interval measurements due to variation and noise in the rise time of pulses and the trigger levels of the counter. The uncertainty components from this jitter can be estimated from the manufacturer specifications or by measuring the TDev for the counter. This is done by connecting the same signal to the start and stop channel of the counter using a long enough cable to give a measurable time interval reading and obtaining values to determine TDev as a function of averaging time. For an averaging time of 10 seconds, the MSL counters have a standard uncertainty of 0.8 ns.

Counter Resolution

The counter resolution is taken directly from the manufacturer specifications. The counter used at MSL has a resolution of 0.7 ns.

Variation in the TV Signal Delay

This uncertainty has been estimated by calculating the TDev over a day between the TV signal and UTC(MSL). A standard uncertainty of 1 ns has been estimated.

Traceability to UTC

MSL is continually comparing the 1pps signal that is designated as UTC(MSL) with the time on each individual Global Positioning System (GPS) satellite. The difference (averaged over five days) between UTC(MSL) and UTC is reported in the BIPM's "Circular T" bulletin. An Allan variance of this data allows the uncertainty at 10 s to be determined.

Component	Distribution	Standard Uncertainty
Time Interval comparison noise	Normal	0.8 ns
Counter Resolution	Rectangular	0.4 ns
Variation in the delay of the television signal	Normal	1 ns
Extrapolation of traceability to UTC	Normal	0.049 ns
Combined Standard Uncertainty		**1.3 ns**
Expanded Uncertainty		**2.6 ns**

The Remote Uncertainties

The remote uncertainties must be estimated at the remote site with the particulars of the counter used there. Only the first three terms in the table above must be estimated. The third term may be taken as 1 ns if no local measurements have been made.

The Remote Frequency Uncertainty

The relative frequency of the remote clock is determined from:

$$f_{\text{rel}} = \left[(t2_{\text{remote}} - t2_{\text{MSL}}) - (t1_{\text{remote}} - t1_{\text{MSL}}) \right] / 86400 \ ,$$

where $t1$ and $t2$ are the time interval measurements at MSL and the remote site on successive days.

The uncertainty for each relative frequency measurement is, therefore:

$$U = \sqrt{(2U_{\text{MSL}})^2 + (2U_{\text{remote}})^2} \Big/ 86400 \ .$$

For example, in the measurement above and assuming that the remote uncertainty is the same as the MSL uncertainty, the relative frequency is reported as $f_{\mathrm{rel}} = (4.8 \pm 6) \times 10^{-14}$ with a coverage factor of k = 2.

Using the Uncertainty

The result above is only for one single measurement of the frequency and does not reflect the uncertainty associated with the behaviour of the remote clock. This needs to be determined by collecting sufficient data to determine either the maximum- minimum behaviour or the Allan deviation of the clock over a longer period of time. For the least uncertainty, the Allan deviation will give a smaller number but requires a more complicated analysis.

References

[1] "Guide to the Expression of Uncertainty in Measurement"

[2] Alavar - http://yazdiet.club.fr/alavar.html.